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Theory of Auger upconversion in quantum wells in a quantizing magnetic field

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Abstract. Scattering rates of Auger and electron–phonon processes between discrete Landau levels in a quasi-two-dimensional electron system are calculated. Using these rates we provide an understanding of the upconversion observed in the magneto-luminescence of one-side modulation doped GaAs/AlGaAs quantum wells; in particular, of its dependence on excitation power and magnetic field.

1. Introduction

Auger scattering in semiconductors is well known from investigations of non-radiative recombination [1]. Free electrons and holes are a prerequisite for this process: the energy obtained in the recombination of an electron–hole pair is taken to excite another electron. The latter electron may lose its excess energy by electron–lattice relaxation; thus the recombination energy is converted into heat. More recently two-particle correlation effects in the Auger process [2] have been demonstrated experimentally for p-GaAs and n-Si [3]. In quantum-well structures Auger processes become possible between different subbands and have been investigated both in theory and experiment (see [4] and references therein). The reduction of Auger-scattering rate in quantum dots due to the discreteness of the electronic states has been used as an argument to propose quantum-dot lasers [5]. Except for an early study of transport in crossed electric and magnetic fields [6], the theory of Auger scattering has been restricted so far to the magnetic-field-free case.

In recent magneto-luminescence experiments by Potemski *et al* [7,8] on one-side modulation doped GaAs/AlGaAs quantum wells an upconversion has been observed and interpreted as being due to an Auger process. These authors studied in photoluminescence and photoluminescence under excitation an asymmetric GaAs/AlGaAs single quantum well of width $d = 25$ nm with an electron density of $N_s = 7.6 \times 10^{11}$ cm^{-2} with a magnetic field $\boldsymbol{B}$ applied in the growth direction. The characteristic energy-level scheme for 7.9 T $\leqslant B \leqslant$ 12.9 T is depicted as the inset in figure 1: the lowest Landau level of the second electric subband L_0' lies between the second L_1 and third L_2 Landau levels of the lowest electric subband (the index refers to the Landau quantum number), while the Fermi energy is pinned to the level L_1. Changing the magnetic field in this interval allows us to tune L_0'

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between L_1 and L_2. The luminescence spectrum under resonant interband excitation into L_1 at low temperature ($T = 1.8$ K) and for low excitation power ($P_{exc} \leqslant 10$ W cm^{-2}) shows two peaks: besides the luminescence due to recombination of an electron from L_0 with a hole in a valence band, a second peak is observed above the exciting laser energy and is related to L_0', i.e. recombination of an upconverted electron with a hole. In order to explain this second peak Potemski *et al* [7, 8] supposed the following processes: after interband excitation of electrons into the partially filled level L_1 (i), recombination takes place between electrons from L_0 and photo-induced holes (ii), then in an Auger process two electrons in L_1 are scattered to L_2 and L_0 (iii) and a relaxation process brings the electron from L_2 to L_0' (iv), from where it recombines with a photo-induced hole (v) to give the upconverted luminescence or, emitting a phonon, relaxes into the level L_1 (vi). The authors also present results of the dependence of the luminescence intensity related to L_0 and L_0' on magnetic field and excitation power. The most surprising result is the high intensity of the L_0' luminescence which can be of the same order as the L_0' luminescence.

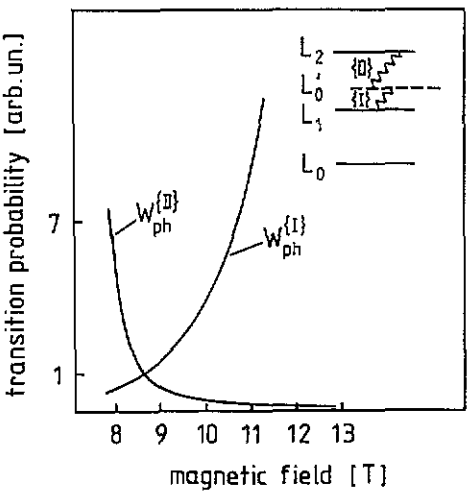

Figure 1. Magnetic field dependence of the probabilities for the transitions between Landau levels $L_0' \rightarrow L_1(\{I\})$ and $L_2 \rightarrow L_0'(\{II\})$ caused by the phonon emission.

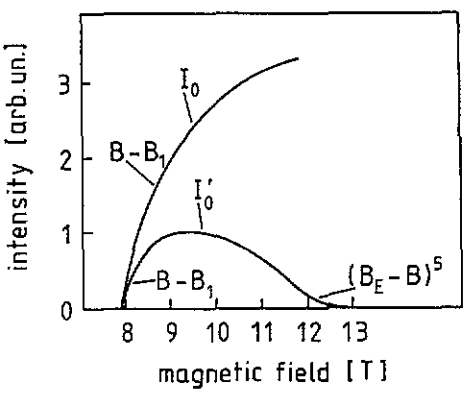

Figure 2. Magnetic field dependence of the luminescence intensities I_0 and I_0' for recombination from the Landau levels L_0 and L_0', respectively.

As already mentioned, theoretical studies of Auger scattering between Landau levels in a 2DEG are missing from the literature.

In the last few years electron–acoustic phonon interaction in a 2DEG in a quantizing magnetic field has attracted attention because of its role in the breakdown of the dissipationless quantum Hall effect [9, 10] and the cooling of a 2DEG at low temperatures $T \leqslant 40$ K [11–13]. The emission and absorption of the ballistic phonon pulses by a 2DEG in MOS structures and GaAs heterostructures was studied in theoretical works [14, 15] as well as in many experiments (see the review paper [16]). In all calculations [9–15, 17], however, only acoustic-phonon-assisted transitions within Landau levels of the lowest subband are considered.

Therefore, we calculate the characteristic times of processes (iii) and (iv), (vi), i.e. Auger scattering between Landau levels of the lowest electric subband and electron–acoustic-phonon scattering between Landau levels of the two lowest electric subbands ($L_2 \rightarrow L_0'$

and $L_0' \rightarrow L_1$), as well as the lifetime of a test hole in level L_0 with respect to both the Auger process and the phonon emission. By analysing rate equations for the processes (i)–(vi) we find an estimate for the time of the Auger process as well as magnetic field and excitation power dependences of the two luminescence peaks that are consistent with the experimental findings.

2. Auger scattering between Landau levels

Due to the combined effect of quantum-well confinement and Landau quantization the energy spectrum of electrons in a quantum well with magnetic field in the growth direction (parallel to z) is discrete. The single-particle energies E_{nl} are characterized by a subband index n and a Landau-level index l (the spin degeneracy is not removed and is taken into account in the occupation factors). The corresponding wave functions

$$\Psi_{nlk}(\boldsymbol{r}) = \chi_{lk}(\boldsymbol{R})\xi_n(z) \qquad \boldsymbol{r} = (\boldsymbol{R}, z) \tag{1}$$

factorize into an oscillator function χ_{lk} for the Landau oscillator and a subband function ξ_n. The single-particle energy does not depend on the quantum number k which results from the asymmetric gauge of the vector potential and counts the degeneracy of the Landau levels.

The scattering time for a single electron due to the Auger process, in which two electrons are scattered from single-particle states 1, 2 into states $1', 2'$, is given by

$$\frac{1}{\tau^{e}_{\text{Auger}}} = \sum_{k_2,k_1',k_2'} W_{1,2\rightarrow 1',2'} f_2(1-f_{1'})(1-f_{2'}) \tag{2}$$

where the occupation probabilities f take into account the Pauli exclusion principle. In detail the transition probability is given by

$$W_{1,2\rightarrow 1',2'} = (2\pi/\hbar)\left|M_{1,2\rightarrow 1',2'}\right|^2 \delta(E_{n_1 l_1} + E_{n_2 l_2} - E_{n_1' l_1'} - E_{n_2' l_2'}). \tag{3}$$

In equation (2) we have taken the sum over the momenta k to account for all equivalent scattering processes, that are possible due to the degeneracy for a given set of Landau levels. After the summation the result does not depend on k_1 (and thus on the choice of the gauge, as it should do). Thus τ^{e}_{Auger} is the relaxation time of a test electron from the level L_1 to L_0 with respect to the Auger process. For later considerations we introduce also the lifetime of a test hole in the level L_0 with respect to the Auger process from the level L_1 as

$$\frac{1}{\tau^{h}_{\text{Auger}}} = \sum_{k_1,k_2,k_2'} W_{1,2\rightarrow 1',2'} f_1 f_2(1-f_{2'}) \tag{4}$$

where $1 - f_2'$ is the occupation probability of a hole in L_2.

The matrix element $M_{1,2\rightarrow 1',2'}$ to be calculated with functions of equation (1) is that of the Coulomb interaction potential after 2D Fourier transformation:

$$V_{ee}(r_1, r_2) = \frac{1}{A}\sum_{q} \frac{2\pi e^2}{\kappa_0 q} \exp\left[iq(\boldsymbol{R}_1 - \boldsymbol{R}_2) - q\,|\,z_1 - z_2\,|\right] \tag{5}$$

where A is the normalization area in the (x, y) plane, and κ_0 is the low-frequency dielectric constant. The screening due to free carriers in the electron subbands is not considered.

In our special case we are interested in transitions only within the first electric subband ($n = 1$). Therefore, using the form factor

$$F(q) = \int dz_1\, dz_2\, \xi_1^2(z_1)\xi_1^2(z_2) \exp\bigl(-q \mid z_1 - z_2 \mid\bigr) \tag{6}$$

the matrix element can be represented in the form

$$M_{1,2\to 1',2'} = \frac{1}{A}\sum_q \frac{2\pi e^2}{\kappa_0 q} F(q) Q_{l_1 l_1'}(q) Q_{l_2 l_2'}(q) \delta_{k_1', k_1 + q_x} \delta_{k_2', k_2 - q_x}$$

$$\times \exp[\tfrac{1}{2} i a_B^2 q_y (k_1 + k_1' - k_2 - k_2') - i\phi(l_1' + l_2' - l_1 - l_2)]. \tag{7}$$

Here $a_B = (\hbar/\mid e \mid B)^{1/2}$ is the magnetic length, and ϕ is the polar angle of the vector q. The functions $Q_{ll'}$ are expressed in terms of the Laguerre polynomials L_l^m [18]: for $l \geqslant l'$

$$Q_{ll'}(q) = (l'!/l!)^{1/2} \exp\bigl(-t/2\bigr) t^{(l-l')/2} L_{l'}^{l-l'}(t) \qquad t = \tfrac{1}{2} q^2 a_B^2$$

and for $l \leqslant l'$

$$Q_{ll'} = (-1)^{l-l'} Q_{l'l}. \tag{8}$$

It is easy to see from (2)–(4) that the momenta k appear only in the matrix element so that using (7) one can find

$$\sum_{k_2, k_1', k_2'} \bigl|M_{1,2\to 1',2'}\bigr|^2 = \frac{1}{A}\frac{1}{2\pi a_B^2}\frac{4\pi^2 e^4}{\kappa_0^2}\sum_q \frac{1}{q^2} F^2(q) Q_{l_1 l_1'}^2(q) Q_{l_2 l_2'}^2(q). \tag{9}$$

It should be noted that after performing the summation of the modulus squared of the matrix element over the momenta k no interference between different Fourier components of the Coulomb potential occurs.

So far we have not considered the δ function in equation (3), which would give a factor of infinity because of energy conservation. It is known for a realistic 2DEG that impurity scattering results in a broadening of the Landau levels [19]. The same mechanism also limits the mobility of the carriers in the system. Therefore, we replace the δ function by a Lorentzian with a width corresponding to the scattering time $\tau \simeq 10$ ps for the mobility $\mu = 2.5\times 10^5$ V cm^2 s^{-2}, which means $\pi\delta(E_{n_1 l_1} + E_{n_2 l_2} - E_{n_1' l_1'} - E_{n_2' l_2'}) \to \tau$. This approach is inconsistent in so far as impurity scattering, in principle, gives k-dependent energies; however, we perform the k sum without taking this into account.

Now from (2), (3) and (9) we obtain

$$1/\tau_{\text{Auger}}^e = W_{\text{Auger}} f_2(1 - f_{1'})(1 - f_{2'})\Big|_{l_1 + l_2 = l_{1'} + l_{2'}} \tag{10}$$

and the probability of the Auger process is

$$W_{\text{Auger}} = 2\bigl(e^2/\hbar\bigr)^2 \Bigl[\bigl(\tau/\kappa_0^2\bigr) a_B^2\Bigr] \Phi\bigl(a_B/z_0\bigr). \tag{11}$$

The overlap integral

$$\Phi\left(\frac{a_B}{z_0}\right) = \int_0^\infty \frac{dq}{q} F^2(q) Q_{l_1 l_1'}^2(q) Q_{l_2 l_2'}^2(q)\Big|_{l_1 + l_2 = l_{1'} + l_{2'}} \tag{12}$$

depends on magnetic field via the dimensionless parameter a_B/z_0 where z_0 is a characterizing length parameter of the lowest electric subband. We have calculated the form factor $\Phi(a_B/z_0)$ for the Auger process involving Landau levels $l_1 = l_2 = 1$ and $l_1' = 0$, $l_2' = 2$ (the Auger process from level L_1 into the levels L_0 and L_2) with the model wavefunction [20, 21]

$$\xi(z) = \sqrt{\tfrac{1}{2}b^3 z}\exp(-\tfrac{1}{2}bz). \tag{13}$$

The value of the parameter $z_0 = 3/b = 10.5$ nm has been chosen to reproduce the separation of the two lowest subbands of the actual quantum well in [7] and [8] by a triangular potential model.

In the range of magnetic fields between 7.9 T and 12.9 T the probability W_{Auger}^{-1} is a slowly decreasing function of B with values between 4.5 and 3.5 fs. However, the occupation factors, which have to be included in order to obtain $\tau_{\text{Auger}}^{\text{e}}$, will drastically increase this value. For the case under consideration in the experiment of [7] and [8] (with $N_s = 7.6\ \text{cm}^{-2}$, $T = 1.8$ K and $B = 9.5$ T, but without pumping) we find by including the occupation factors $\tau_{\text{Auger}}^{\text{e}} \simeq 10^{63} W_{\text{Auger}}^{-1}$, i.e. the Auger process is not possible at all because the lower Landau level L_0 is almost completely filled. It becomes possible only by optical pumping into the level L_1 and subsequent recombination from L_0, thus creating the empty states required for the Auger process. Without pumping the Auger process is possible only at much lower magnetic fields (for lower carrier density) or much higher temperatures.

Applying the same considerations to $\tau_{\text{Auger}}^{\text{h}}$ we obtain an expression similar to (10) but with the factor f_1 instead of the factor $1 - f_1'$. This gives for half-filled L_1 level $\tau_{\text{Auger}}^{\text{h}} \simeq 2$ fs. In contrast to $\tau_{\text{Auger}}^{\text{e}}$ we see that $\tau_{\text{Auger}}^{\text{h}}$ does not depend on available free places in the level L_0 and shows the efficiency of the Auger process in comparison with other processes which add (by emission of phonons) or remove (by interband recombination) electrons in L_0.

3. Electron–acoustic-phonon scattering between Landau levels

Transitions between Landau levels of the different electric subbands are possible by emission of acoustic phonons via the deformation (DA) potential interaction. The corresponding relaxation time of a test electron is given by

$$\frac{1}{\tau_{nl\to n'l'}^{\text{DA}}} = (1 - f_{n'l'}) \sum_{k'} W_{nlk\to n'l'k'}^{\text{DA}} \tag{14}$$

where the transition probability from the state Ψ_{nlk} to the state $\Psi_{n'l'k'}$ is given by [17]

$$W_{nlk\to n'l'k'}^{\text{DA}} = \frac{1}{A}\frac{\Xi^2}{\hbar\rho s^2}\frac{\omega^2}{s^2}\sum_q \frac{1}{a}\delta_{k',k+q_x} Q_{ll'}^2(q)\,|I_{nn'}(a)|^2 \tag{15}$$

with the form factor

$$I_{nn'}(a) = \int \mathrm{d}z\, \xi_n(z)\xi_{n'}(z)\exp(\mathrm{i}az) \qquad a = \sqrt{\frac{\omega^2}{s^2} - q^2}. \tag{16}$$

Here Ξ is the deformation potential constant, ρ is the mass density of the crystal, s is the velocity of sound and the transferred energy $\omega = E_{nl} - E_{n'l'}$.

Direct calculation shows that the probability for the piezoelectric potential (PA) may be obtained from (15) by replacing $\Xi^2(\omega^2/s^2)$ by $(e\beta)^2$ where β is the piezoelectric modulus of the crystal averaged over directions of propagation of the phonon and its polarizations. Therefore, for GaAs one can find

$$W^{\mathrm{DA}}/W^{\mathrm{PA}} = \Xi^2(\omega^2/s^2)/(e\beta)^2 = \left[\omega\ (\mathrm{meV})/0.42\right]^2. \tag{17}$$

This means that in strong quantizing magnetic fields ($B \simeq 10$ T) when L_0' is sufficiently far away from L_1 and L_2 the PA interaction is suppressed with respect to the DA interaction.

From (15), taking a sum over the momentum k' of the final states, it is easy to find

$$W_{nl\to n'l'} = \sum_{k'} W^{\mathrm{DA}}_{nlk\to n'l'k'} = \frac{\Xi^2}{2\pi\hbar\rho s^2}\frac{\omega^2}{s^2}\int \mathrm{d}z_1\,\mathrm{d}z_2\,\xi_{n_1}(z_1)\xi_{n_2}(z_1)\xi_{n_1}(z_2)\xi_{n_2}(z_2) \times K\left(\frac{\omega}{s}\mid z_1 - z_2\right) \tag{18}$$

where the kernel

$$K\left(\frac{\omega}{s}\mid z_1 - z_2\right) = \int_0^{\omega/s} \mathrm{d}q\,\frac{q}{a}\,Q^2_{ll'}(q)\exp\left[ia(z_1 - z_2)\right]. \tag{19}$$

The following two transitions will be considered:

$$n = 2 \qquad l = 0 \to n' = 1 \qquad l' = 1(L_0' \to L_1) \tag{I}$$

with

$$\omega = \omega_1 = E_{20} - E_{11} = \omega_E - \omega_B \qquad Q^2_{10}(q) = t\exp(-t)$$

and

$$n = 1 \qquad l = 2 \to n' = 2 \qquad l' = 0(L_2 \to L_0') \tag{II}$$

with

$$\omega = \omega_1 = E_{12} - E_{20} = 2\omega_B - \omega_E \qquad Q^2_{20}(q) = (t^2/2)\exp(-t).$$

Here $\omega_B = eB/mc$ is the cyclotron energy and $\omega_E = E_{20} - E_{10}$ is the energy separation between first and second electric subbands.

The kernel (19) for these two transitions can be rewritten

$$K\left(\frac{\omega}{s}\mid z_1 - z_2\right)\Big|_{\{\mathrm{I;II}\}} = \frac{1}{a_B^2}\int_0^{\alpha} \mathrm{d}t\,\{t;(t^2/2)\}\frac{1}{a}\,\exp[ia(z_1 - z_2) - t] \tag{20}$$

where $\alpha = (\omega^2/2s^2)a_B^2 \simeq \omega_B/2ms^2 \gg 1$ is a large parameter. This integration, evaluated by means of the steepest-descent method, gives

$$K\left(\frac{\omega}{s}\mid z_1 - z_2\right)\Big|_{\{\mathrm{I;II}\}} = \frac{\omega}{s}\frac{1}{2\alpha}\frac{(1-\mathrm{i}\beta)^2}{(1+\beta^2)^2}\left\{1;\frac{1-\mathrm{i}\beta}{1+\beta^2}\right\}\exp\left[\mathrm{i}\frac{\omega}{s}(z_1 - z_2)\right] + \mathrm{o}\left(\frac{1}{\alpha}\right) \tag{21}$$

where the parameter $\beta = (\omega/s)(z_1 - z_2)/2\alpha$. For $\omega \simeq \omega_B$ and for $z_1, z_2 \simeq d$ we have $\beta \simeq ms(z_1 - z_2) \sim msd$. For GaAs the length $\hbar/ms = 340$ nm so that at $d = 25$ nm we have $\beta \ll 1$. Hence, neglecting the terms of higher order in β we obtain

$$K\left(\frac{\omega}{s} \mid z_1 - z_2\right)\Big|_{\{\mathrm{I};\mathrm{II}\}} \simeq \frac{\omega}{s}\frac{1}{2\alpha}\exp\left[\mathrm{i}\frac{\omega}{s}(z_1 - z_2)\right]. \tag{22}$$

One can see that in this approximation the kernel differs for the two transitions only by the different values of the transferred energy ω. This result for K is easy to find from (19) by replacing a by ω/s. This means that in a 2DEG in a quantizing magnetic field normal to the electron sheet, electrons interact mainly with phonons which propagate in the direction of $\boldsymbol{B}$. It should be noted that this statement is true not only for these particular transitions but also in general.

Substituting the kernel (22) in (18) we obtain the probability for the transitions {I} and {II}

$$W_{\mathrm{ph}}^{\{\mathrm{I};\mathrm{II}\}} = \left(1/\bar{\tau}_B\right)(\omega/s)a_B\,|I_{12}(\omega/s)|^2 \qquad \omega = \{\omega_1; \omega_2\}. \tag{23}$$

Here we define a nominal interaction time

$$1/\bar{\tau}_B = \Xi^2/2\pi\hbar\rho s^2 a_B^3. \tag{24}$$

Since the parameter $(\omega/s)d \gg 1$ (in fact, this defines the energy range in which we are interested), the form factor I_{12} can be calculated by the method of a stationary phase [22]. Using Airy functions instead of wave functions $\xi_n(z)$ one can find

$$|I_{12}(\omega/s)|^2 = 4[(\omega/s)\bar{z}]^{-6} \qquad \bar{z} = \hbar/\sqrt{2m\omega_E/(\alpha_1 - \alpha_0)} \tag{25}$$

where α_i is the ith zero of the Airy function.

Substituting (25) in (23) we find

$$W_{\mathrm{ph}}^{\{\mathrm{I}\}} = \left(1/\bar{\tau}_B\right)\left(a_B/\bar{z}\right)4(s/\bar{z})^5/(\omega_E - \omega_B)^5 \propto B(B_E - B)^{-5} \tag{26}$$

$$W_{\mathrm{ph}}^{\{\mathrm{II}\}} = \left(1/\bar{\tau}_B\right)\left(a_B/\bar{z}\right)4(s/\bar{z})^5/(2\omega_B - \omega_E)^5 \propto B(2B - B_E)^{-5} \tag{27}$$

where B_E is the magnetic field when $\omega_B = \omega_E$. For GaAs at $B = 9$.T and $\bar{z} = 6.7$ nm (this corresponds to $\omega_E = 22.4$ meV taken from [7] and [8]) we find $W_{\mathrm{ph}}^{\{\mathrm{II}\}} = (32.3\ \mu\mathrm{s})^{-1}$ and $W_{\mathrm{ph}}^{\{\mathrm{I}\}} = (1.5\ \mu\mathrm{s})^{-1}$. Note that the relaxation probability calculated in [17] for magnetic fields $B \simeq 1$ T is much greater. Such a suppression of the electron–phonon interaction at $\omega \simeq \omega_B$ and $(\omega_B/s)\bar{z} \gg 1$ follows from the conservation laws. The electron states in this regime constitute a wave packet so that the states with $l, n \simeq 1$ have momenta of the order of a_B^{-1} in the (x, y) plane and of the order of $\bar{z}^{-1}$ in the z direction. Therefore, only for a small number of electron states in this packet is the momentum conservation law fulfilled at the interaction with acoustic phonons with momenta $\omega/s \gg a_B^{-1}, \bar{z}^{-1}$.

The dependence of the probabilities (26) and (27) on the magnetic fields is plotted in figure 1. At low fields $B \simeq 8$ T the transition {II} ($L_2 \rightarrow L_0'$) is predominant. As B increases, the probability $W_{\mathrm{ph}}^{\{\mathrm{II}\}}$ rapidly falls while $W_{\mathrm{ph}}^{\{\mathrm{I}\}}$ (this corresponds to $L_0' \rightarrow L_1$) slowly increases so that already at $B = 8.6$ T these two transitions are equally probable. At high fields the transition $L_2 \rightarrow L_0'$ is suppressed with respect to $L_0' \rightarrow L_1$ and $W_{\mathrm{ph}}^{\{\mathrm{I}\}}$

rapidly increases with increasing B so that at fields near B_E it reaches values corresponding to times less than 1 ns.

In order to obtain the relaxation times $\tau_{ph}^{\{I\},\{II\}}$, the occupation factors have to be included according to equation (14). Noting that for the case under consideration in experiment [7, 8] the levels L_1, L_0' are not fully occupied, we obtain

$$\tau_{ph}^{\{I\},\{II\}} = (1 - f_{11,20}) W_{ph}^{\{I\},\{II\}} \simeq W_{ph}^{\{I\},\{II\}}. \tag{28}$$

To find out which mechanism (the Auger scattering or the phonon emission) is responsible for filling of holes in the level L_0 with electrons, we also estimate the lifetime of a test hole in L_0 with respect to the phonon emission from the higher level L_1:

$$\frac{1}{\tau_{ph}^{h}} = \sum_{k'} W_{11k \to 10k'}^{DA} f_{11} \equiv W_{1 \to 0} f_{11}. \tag{29}$$

Here the probability $W_{11k \to 10k'}^{DA}$ is given by equation (15) for the transferred energy $\omega = \omega_B$. Since $\omega_B > \omega_1, \omega_2$ always, the approximations that have been made above to calculate the kernel K and form factor $I_{nn'}$ are also justified for this case. Therefore one can obtain $W_{1 \to 0}$ from equation (22) by substituting $\omega = \omega_B$. It is clear that the lifetime of a test hole τ_{ph}^{h} is larger than the relaxation times $\tau_{ph}^{\{I\},\{II\}} \simeq 1 \mu s$ and thus much larger than $\tau_{Auger}^{h} (\simeq 1$ fs). Therefore, the Auger process is much more efficient to fill holes in the level L_0 than the phonon emission. This fact makes the observation of the Auger upconversion by interband optical pumping possible [7, 8].

4. Analysis of the rate equations

So far we have considered only the processes (iii), (iv) and (vi). In order to correctly describe the experimental situation of [7] and [8] we have to take into account also the processes (i), (ii) and (v), i.e. the pumping by interband excitation to the level L_1 and the recombination of electrons from L_0 and L_0' with the photo-induced holes. As has been discussed already in section 2, the Auger process becomes possible only after processes (i) and (ii). Intensities of the emissions from the levels L_0 and L_0' are determined by the characteristic times of the processes (i)–(vi) from the following set of rate equations:

$$\partial n_0/\partial t = n_0/\tau_0 - n_1/\tau_{Auger}^{e} \tag{30a}$$

$$\partial n_1/\partial t = \tilde{P}_{exc} + n_0'/\tau_{ph}^{\{I\}} - 2n_1/\tau_{Auger}^{e} \tag{30b}$$

$$\partial n_0'/\partial t = n_2/\tau_{ph}^{\{II\}} - n_0'/\tau_{ph}^{\{I\}} - n_0'/\tau_0' \tag{30c}$$

$$\partial n_2/\partial t = n_1/\tau_{Auger}^{e} - n_2/\tau_{ph}^{\{II\}}. \tag{30d}$$

Here n_l (n_l') is the areal number density of electrons in the Landau level with index l of the first (second) electric subband including spin degeneracy. The characteristic times of the recombination of electrons from level L_0 and L_0' with photo-induced holes are τ_0 and τ_0' which depend on B in the same way and are of the same order of magnitude. In equation (30b) we have also defined the flux of electrons created by interband excitation into the level L_1

$$\tilde{P}_{exc} = (P_{exc}/\omega_{exc})(1 - f_1) \tag{31}$$

where P_{exc} and ω_{exc} are the excitation power and energy. The factor $1-f_1$ takes into account the availability of free states in the level L_1. In a stationary case we have $\partial n/\partial t = 0$ and the rate equations reduce to

$$I_0 \equiv n_0/\tau_0 = n_1/\tau^{e}_{Auger} = n_2/\tau^{[II]}_{ph} \tag{32a}$$

$$I_0' \equiv n_0'/\tau_0' \tag{32b}$$

$$I_0 - I_0' = n_0'/\tau^{[I]}_{ph} \tag{32c}$$

$$I_0 + I_0' = \tilde{P}_{exc} \tag{32d}$$

where I_0 and I_0' are proportional to the luminescence intensities for recombinations from L_0 and L_0', respectively. According to equation (32*a*) the luminescence intensity I_0 is determined both by τ^{e}_{Auger} and $\tau^{[II]}_{ph}$. Because $\tau^{e}_{Auger} = (n_1/n_2)\tau^{[II]}_{ph} > \tau^{[II]}_{ph}$ we find for B = 9.5 T that $\tau^{e}_{Auger} > 30$ μs. This means that the relaxation time of a test electron in the level L_1 with respect to the Auger scattering is strongly enhanced due to the occupation factors over the characteristic time W^{-1}_{Auger} and the lifetime of a test hole τ^{h}_{Auger} in the level L_0, which are of the order of 1 fs.

Because τ^{h}_{Auger} is much smaller than all other characteristic times of this system, the Auger-upconversion mechanism immediately fills all holes arising in L_0 due to the recombination of electrons from L_0 and creates electrons in the level L_2. Thus the number of upconverted electrons is always equal to the number of recombining electrons from L_0. However, the intensity of the upconverted luminescence I_0' is determined by the competition of the processes (v) and (iv), i.e. by the ratio of τ_0' and $\tau^{[I]}_{ph}$ (see (32*b*) and (*c*)).

In order to obtain the dependences of I_0 and I_0' on the magnetic field and the excitation power we consider two different cases. In a case when process (v) is dominant over the process (vi), i.e. $\tau^{[I]}_{ph} \gg \tau_0'$, we find from equation (32) that

$$I_0 \simeq I_0' \simeq \tfrac{1}{2}\tilde{P}_{exc} \gg N_0/\tau^{[I]}_{ph} \qquad n_0 \simeq n_0' \simeq N_0 \tag{33a}$$

which means that

$$I_0, I_0' \propto P_{exc}, (B - B_1)/B. \tag{33b}$$

Here N_0 is the capacity of one Landau level and B_1 corresponds to the full occupation of two Landau levels L_0 and L_1. With the help of (26), (28) and (31) it is seen that the inequality in equation (33), which is determined by $\tau^{[I]}_{ph} \gg \tau_0'$, corresponds to a situation when the magnetic field is close to the lower bound of the considered interval and at the same time the excitation power is high. In the opposite case when $\tau^{[I]}_{ph} \ll \tau_0'$, which corresponds to magnetic fields close to the upper bound and low powers, we find from (32)

$$I_0' = \tilde{P}^2_{exc}/\big(N_0/\tau_{ph}[I]\big) \ll I_0 = \tilde{P}_{exc} \qquad n_0 \simeq N_0 \gg n_0'. \tag{34}$$

This means that

$$I_0' \propto P^2_{exc}, \big[(B - B_1)/B\big](B_E - B)^5 \tag{35}$$

while

$$I_0 \propto P_{exc}, (B - B_1)/B. \tag{36}$$

Now it is clear from equation (33) that at B near B_1 both intensities I_0, I_0' increase linearly with B. As B increases further, I_0 continues to increase in accordance with (36) but not so sharply as near B_1. However, I_0' shows another behaviour given by equation 35 and decreases with B as $(B_E - B)^5$ when B is near B_E (figure 2). At low powers I_0' depends quadratically on P_{exc} (equation (35)) while at high powers for I_0' (equation (33)) and in the whole range of the power variation for I_0 ((33), (36)) this dependence is linear.

In conclusion, our calculation provides an understanding of the main features of the experiments reported in [7] and [8]. In the light of our investigations it would be interesting to have detailed experimental information on the intensities I_0 near B_1 and I_0' near B_E for which our theory provides definite information.

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